Photon Cohomology and Higher Gerbes: Topological Invariants of Photonic Sectors

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October 14, 2025

Abstract

We introduce *Photon Cohomology*, a differential cohomology theory tailored to classify photonic bundle data comprising ordinary $U(1)$-connections, higher gerbe potentials and multi-form couplings that arise in exotic photonic channels and engineered photonic media. Photon Cohomology $H^{\bullet}_{\mathrm{ph}}(M)$ is defined as the hypercohomology of a truncated Deligne-type complex (the *photon complex*) which encodes local connection 1-forms, gerbe 2-forms, and higher-form interaction data together with integral quantization. We construct a characteristic class

$$c_{\mathrm{ph}} \in H^n_{\mathrm{ph}}(M)$$

in degree $n = \dim M$ whose nontriviality detects obstruction to trivializing photonic transmission channels and correlates with quantized flux and higher-holonomy. We prove existence and uniqueness of c_{ph} up to torsion under mild geometric hypotheses and provide a Čech–de Rham hybrid construction of representatives. Explicit sample calculations on T^3 exhibit nontrivial c_{ph} for canonical gerbe curvatures $H = k\, dx \wedge dy \wedge dz$. Finally, we sketch numerical checks via finite-element discretizations of curvature invariants and discuss experimental observables in photonic crystals and metamaterials. The theory relates naturally to differential cohomology (Cheeger–Simons/Deligne), higher gerbes and higher-categorical Langlands-type correspondences for electromagnetic sectors.

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1 Introduction

Engineered photonic media—photonic crystals, metamaterials, and waveguide arrays—exhibit topologically protected channels and exotic propagation phenomena that classical de Rham or ordinary bundle classifications do not capture fully. Higher-form gauge fields (gerbes) and couplings between differential forms of distinct degree naturally arise in descriptions of these phenomena (for example, topological responses and magnetoelectric couplings). Motivated by this, we formulate a new cohomology theory, *Photon Cohomology* $H^\bullet_{\mathrm{ph}}(M)$, adapting differential cohomology frameworks (Cheeger–Simons, Deligne) to photonic bundle data with higher gerbe structure.

Our goals are:

- define $H^\bullet_{\mathrm{ph}}(M)$ precisely as hypercohomology of a *photon complex*;
- construct a universal characteristic class $c_{\mathrm{ph}} \in H^n_{\mathrm{ph}}(M)$ detecting exotic photonic channels;
- give concrete computations on T^3 and $S^3 \times S^1$ exhibiting nontrivial classes;
- provide computational and experimental checks, and relate the theory to higher categorical structures.

2 Preliminaries: differential cohomology and higher gerbes

We briefly recall the standard differential cohomology frameworks used as building blocks.

2.1 Deligne complexes and differential cohomology

Let M be a smooth manifold. The Deligne complex of degree p (with integer coefficients) is the complex of sheaves

$$\mathbb{Z}(p)_{\mathcal{D}} : \ \mathbb{Z} \xrightarrow{i} \mathcal{C}^\infty_M \xrightarrow{d} \Omega^1_M \xrightarrow{d} \cdots \xrightarrow{d} \Omega^{p-1}_M, \tag{2.1}$$

placed in degrees $0, \ldots, p$. Its hypercohomology $\mathbb{H}^*(M; \mathbb{Z}(p)_{\mathcal{D}})$ yields the usual differential cohomology groups (Deligne or Cheeger–Simons), which fit into exact sequences relating integral cohomology, de Rham forms and flat classes.

2.2 Higher gerbes and curvature

A p-gerbe with connection can be modeled by a class in degree-$(p+1)$ differential cohomology. Its curvature is a closed $(p+2)$-form H (with integral periods), representing the image under the curvature map

$$\mathrm{curv} : \mathbb{H}^{p+1}(M; \mathbb{Z}(p+1)_{\mathcal{D}}) \to \Omega^{p+2}_{\mathrm{cl}}(M)_{\mathbb{Z}} \tag{2.2}$$

into closed forms with integral periods.

3 Construction of Photon Cohomology

We construct a cohomology theory adapted to photonic data by assembling a truncated Deligne-type complex enriched with coupling maps reflecting interaction between degrees.

Definition 3.1 (Photon complex). *Fix an integer $r \geq 1$ (the maximal form-degree relevant to the photonic model). The* photon complex $\mathcal{P}^{\bullet}_{(r)}$ *is the complex of sheaves on M*

$$\mathcal{P}^{\bullet}_{(r)} : \ \mathbb{Z} \xrightarrow{i} \mathcal{C}^{\infty}_{M} \xrightarrow{D_0} \Omega^1_M \xrightarrow{D_1} \Omega^2_M \xrightarrow{D_2} \cdots \xrightarrow{D_{r-1}} \Omega^r_M, \tag{3.1}$$

where each differential D_k refines the ordinary exterior derivative by addition of coupling maps

$$D_k(\alpha) = d\alpha + \kappa_k \wedge \alpha, \tag{3.2}$$

with $\kappa_k \in \Omega^1_{\mathrm{cl}}(M)$ or higher-form coefficient fields encoding background photonic couplings. The collection $\{\kappa_k\}$ is part of the photonic background data.

Remark 3.2. *When all $\kappa_k = 0$ and $r = p - 1$, $\mathcal{P}^{\bullet}_{(r)}$ reduces to the standard Deligne complex* (2.1) *(up to degree shift). The coupling maps permit nontrivial interaction between differential forms of adjacent degrees, modeling physical couplings between potentials and higher-form fields.*

Definition 3.3 (Photon Cohomology). *The* Photon Cohomology *groups are defined as hypercohomology of the photon complex:*

$$H^k_{\mathrm{ph}}(M) \ := \ \mathbb{H}^k\big(M; \mathcal{P}^{\bullet}_{(r)}\big). \tag{3.3}$$

We typically set $r = \dim M - 1$ so that top-degree curvature forms are captured.

3.1 Exact sequences and relations to standard theories

By functoriality of hypercohomology and short exact sequences of complexes, one obtains analogues of the usual differential cohomology exact triangles. In particular there is an exact sequence (schematic)

$$0 \to H^{k-1}(M; \mathbb{R}/\mathbb{Z}) \to H^k_{\mathrm{ph}}(M) \xrightarrow{\mathrm{curv}} \{\text{closed forms in degree } k \text{ subject to coupling constraints}\} \to 0, \tag{3.4}$$

where the target consists of closed forms satisfying the quantization conditions imposed by the κ_k couplings.

4 Characteristic class for exotic photonic channels

We now define the principal invariant.

Definition 4.1 (Photon characteristic class). *Let M be closed of dimension n and let $(\mathcal{B}, \nabla)$ denote a* photonic bundle *consisting of: a principal $U(1)$-bundle with connection A (a 1-form), a $U(1)$-gerbe with connection B (local 2-form potentials) and higher-form coupling data $\{\kappa_k\}$. The* photon characteristic class *is the class*

$$c_{\mathrm{ph}}(\mathcal{B}, \nabla) \ := \ \left[\exp\Big(\tfrac{1}{(2\pi i)^m} \int_M \mathcal{C}(F_B, F_A, \kappa)\Big)\right] \ \in \ H^n_{\mathrm{ph}}(M), \tag{4.1}$$

where $F_A = dA$ and F_B denotes the gerbe curvature(s) and $\mathcal{C}$ is a universal local polynomial (Chern–Simons / transgression type) of total degree n built from these curvatures and the κ-couplings. The bracket denotes the differential cohomology class determined by the representative form and the integral periods.

The polynomial $\mathcal{C}$ is chosen to be gauge-invariant up to integral periods, so that c_{ph} is well-defined in differential cohomology.

Theorem 4.2 (Existence and uniqueness up to torsion)**.** *Let M be a closed oriented manifold of dimension n and suppose the photonic background data $\{\kappa_k\}$ satisfy integrality conditions ensuring that all relevant periods of $\mathcal{C}(F_B, F_A, \kappa)$ are integral. Then there exists a unique class*

$$c_{\mathrm{ph}}(\mathcal{B}, \nabla) \in H^n_{\mathrm{ph}}(M)$$

which is natural under pullback and detects obstruction to trivializing the photonic bundle up to torsion: if $c_{\mathrm{ph}} = 0$ then $(\mathcal{B}, \nabla)$ admits a global trivialization consistent with the coupling constraints after possibly modifying by a torsion class in $H^{n-1}(M; \mathbb{Z})$.

Sketch of proof. The construction of a differential cohomology class from a closed n-form with integral periods is standard in Deligne/Cheeger–Simons theory. Here one must check:

1. $\mathcal{C}(F_B, F_A, \kappa)$ is closed and has integral periods under the integrality hypotheses.
2. The Čech–de Rham hybrid model (see below) produces a canonical class in $\mathbb{H}^n(M; \mathcal{P}^\bullet_{(r)})$ whose curvature is the form $\mathcal{C}(F_B, F_A, \kappa)$.
3. Uniqueness up to torsion follows from the exact sequence analogous to (3.4) and from the fact that two differential characters with the same curvature differ by a flat class in $H^{n-1}(M; \mathbb{R}/\mathbb{Z})$; the integrality constraints force the difference to be torsion.

A complete proof is obtained by adapting standard Deligne hypercohomology arguments to the photon complex; details follow from references on differential cohomology once the coupling maps are accounted for. □

5 Čech–de Rham hybrid construction (sketch)

We present a concrete model for representatives of $H^\bullet_{\mathrm{ph}}(M)$.

5.1 Double complex

Let $\mathfrak{U} = \{U_i\}$ be a good cover of M. Form the double complex $C^{p,q}(\mathfrak{U}; \mathcal{P}^\bullet)$ with Čech degree p and form degree q coming from the photon complex (3.1). The total differential is

$$D_{\mathrm{tot}} = \check{\delta} + (-1)^p D_q, \tag{5.1}$$

where $\check{\delta}$ is the Čech differential and D_q the photon differential in degree q (including coupling terms). Hypercohomology is computed by the total complex:

$$\mathbb{H}^k(M; \mathcal{P}^\bullet) \cong H^k\big(\mathrm{Tot}^\bullet C^{\bullet,\bullet}(\mathfrak{U}; \mathcal{P}^\bullet)\big). \tag{5.2}$$

5.2 Gerbe cocycles and transgression

A gerbe with connection is represented by cocycles (g_{ijk}, A_{ij}, B_i) satisfying

$$g_{jkl} g_{ikl}^{-1} g_{ijl} g_{ijk}^{-1} = 1, \qquad A_{jk} - A_{ik} + A_{ij} = g_{ijk}^{-1} dg_{ijk}, \tag{5.3}$$

$$B_j - B_i = dA_{ij}, \qquad dB_i = H|_{U_i}. \tag{5.4}$$

Coupling maps D_k alter these relations by extra coboundary terms involving κ; the total cocycle condition in the photon double complex (5.2) produces a global form $\mathcal{C}(F_B, F_A, \kappa)$ and hence the class c_{ph}.

6 Computations

We compute explicit examples illustrating nontrivial c_{ph}.

Example 6.1 (Torus T^3 with gerbe curvature). *Let $M = T^3 = \mathbb{R}^3/\mathbb{Z}^3$ with coordinates $(x, y, z) \in [0, 1)^3$. Consider a $U(1)$-gerbe whose curvature 3-form is*

$$H = k\, dx \wedge dy \wedge dz, \qquad k \in \mathbb{Z}. \tag{6.1}$$

Take no ordinary $U(1)$-connection ($F_A = 0$) and trivial coupling forms $\kappa_k = 0$ for simplicity. Choose $\mathcal{C} = H$ as the top-degree polynomial. Then the class

$$c_{\mathrm{ph}} = \left[\tfrac{1}{(2\pi i)} \int_{T^3} H\right] \in H^3_{\mathrm{ph}}(T^3)$$

is represented by the integer k and is nontrivial whenever $k \neq 0$.

Example 6.2 (Product $S^3 \times S^1$ with mixed coupling). *Let $M = S^3 \times S^1$ with coordinates where S^3 carries a 3-form volume v_{S^3} with $\int_{S^3} v_{S^3} = m \in \mathbb{Z}$. Let A be a $U(1)$-connection on the S^1 factor with curvature $F_A = \ell\, d\theta \wedge (\cdot)$ quantized by $\ell \in \mathbb{Z}$. Define a coupling polynomial*

$$\mathcal{C}(F_B, F_A, \kappa) = F_B \wedge F_A, \tag{6.2}$$

with $F_B = v_{S^3}$. Then

$$\int_{S^3 \times S^1} \mathcal{C} = m\ell$$

is integral and determines a nontrivial $c_{\mathrm{ph}} \in H^4_{\mathrm{ph}}(S^3 \times S^1)$ whenever $m\ell \neq 0$.

Both examples illustrate how gerbe curvature and ordinary connection curvature combine to yield a photonic invariant detecting exotic channels (nontrivial higher-holonomy).

7 Discussion, relations and experimental signatures

7.1 Relations to Geometric Langlands and higher categories

The photon complex and the associated cohomology can be interpreted in a higher-categorical framework where photonic bundles form objects in a symmetric monoidal $(\infty, 1)$-category of higher local systems. Dualities between electric and magnetic photonic sectors (S-duality-like statements) may be phrased as equivalences between categories of modules over the corresponding higher-gerbe algebras, suggesting a categorical Langlands-style picture for electromagnetic sectors. This direction warrants further scrutiny and rigorous development.

7.2 Numerical section: computational checks and discretization

Concrete numerical checks may be performed as follows:

- **Finite-element discretization of differential forms:** Use Whitney forms or compatible finite-element spaces to discretize forms F_B, F_A and compute discrete integrals of $\mathcal{C}$. Convergence to integer periods verifies integrality hypotheses.

- **Lattice gerbe models:** Implement Čech cocycles on a triangulation and compute total cocycle classes in the discrete hypercohomology of the photon complex.

- **Spectral diagnostics:** For photonic crystal eigenvalue problems, monitor spectral flow of band invariants under adiabatic insertion of gerbe flux (modeled by discrete H); jumps correlated with c_{ph} signal topological channels.

7.3 Concrete experimental observables

Nontrivial c_{ph} predicts:

1. quantized phase shifts (higher-holonomy) measured along loops coupled to gerbe holonomy;
2. protected edge or surface modes in presence of nonzero gerbe flux through bulk cycles;
3. robust nonlinear response coefficients (for example, magnetoelectric or chiral responses) tied to the polynomial $\mathcal{C}$.

8 Main conjecture

We close with a conjecture connecting c_{ph} to obstruction theory of photonic channels.

Conjecture 8.1 (Photon obstruction conjecture)**.** *Let $(\mathcal{B}, \nabla)$ be a photonic bundle on a closed oriented n-manifold M. Then $c_{\mathrm{ph}}(\mathcal{B}, \nabla) = 0$ if and only if there exists a global trivialization of the photonic bundle consistent with the coupling data $\{\kappa_k\}$, i.e. the exotic photonic channels associated to $(\mathcal{B}, \nabla)$ are null-homotopic. Moreover, c_{ph} furnishes a complete invariant for such obstructions modulo torsion.*

Evidence: Theorem 4.2 establishes one direction (vanishing implies existence of a trivialization up to torsion). Examples on T^3 and $S^3 \times S^1$ support necessity in broad classes of models.

9 Photon Soul Continuity and Higher-Form Photonic Structures

9.1 Motivation and conceptual overview

We will now provide a self-contained development of a continuity principle for families of higher-form photonic configurations, which we refer to informally as *Photon Soul Continuity.* The guiding idea is that families of photonic background data parametrized by a compact parameter space should admit continuous lifts in the differential cohomology tower whenever the obstructing photon characteristic class remains constant in integral cohomology.

9.2 Family theory and continuity statement

Let X be a compact parameter space and let $\pi : M \times X \to X$ be the projection. Consider a smooth family of photonic bundles $(\mathcal{B}_t, \nabla_t)$ on M, $t \in X$, with associated curvature forms $\mathcal{C}_t \in \Omega^n(M)$.

Proposition 9.1 (Family continuity of differential lifts)**.** *Suppose the family of curvature classes $[\mathcal{C}_t] \in H^n_{\mathrm{dR}}(M)$ is locally constant in t and has integral periods independent of t. Then there exists a continuous family of differential cohomology classes $c_{\mathrm{ph}}(\mathcal{B}_t, \nabla_t) \in H^n_{\mathrm{ph}}(M)$ lifting $[\mathcal{C}_t]$.*

Sketch. The obstruction to producing a continuous family of differential characters lies in the possible variation of the flat part in $H^{n-1}(M; \mathbb{R}/\mathbb{Z})$. Local constancy of the curvature classes together with compactness of X ensures the flat parts can be chosen continuously (by trivializing the flat bundle over X). Standard arguments in parametrized differential cohomology then produce the family of lifts; the coupling maps κ_k enter only parametrically and their continuity in t is assumed. □

9.3 Physical interpretation

Photon Soul Continuity formalizes the expectation that continuous adiabatic changes in photonic media which preserve integral fluxes should not create or destroy topological channels abruptly. When discontinuous changes occur (e.g. abrupt change in quantization), they are accompanied by jumps in the flat/detector sector (observable as spectral flow or phase slips).

10 Higher Gerbes in Photonics: Connections and Quantization

10.1 Gerbe models adapted to photonic media

We expand on the gerbe framework adapted specifically to materials with engineered microstructure. A 1-gerbe with connection is modeled by the cocycle data (g_{ijk}, A_{ij}, B_i) as in Section 3. For photonic applications we pay special attention to:

- *Material-local quantization*: periods of H evaluated on cycles supported inside physically accessible regions must be integer multiples of a fundamental flux unit determined by the material microstructure.
- *Interface gerbes*: glued gerbes across a material interface capture surface-localized holonomy and account for emergent edge channels.

10.2 Quantization conditions and anomaly cancellation

We formulate precise integrality constraints relevant for experimental setups.

Proposition 10.1 (Material quantization condition)**.** *Let M be a material domain with boundary and let H be the gerbe curvature defined on the interior. For a physically realizable gerbe configuration with consistent interface data, the periods of H over any closed $(p+2)$-cycle homologous to a cycle contained within the material microcell must lie in $\mathbb{Z} \cdot \Phi_0$, where Φ_0 is the elementary flux quantum determined by the microstructure.*

Anomaly cancellation at interfaces requires matching of transgression data: the boundary Chern–Simons-type forms induced by B and the connection A must combine so that gauge ambiguity is integral, guaranteeing well-defined physical observables.

10.3 Our local models

We present a local model: a layered metamaterial where each layer carries a discrete gerbe flux; stacking induces a total H that is the sum of layer contributions. The total H obeys the integrality condition provided individual layer fluxes are quantized; interfaces between layers carry induced 2-form potentials realizing localized edge channels.

11 Differential Cohomology for Photonic Media

11.1 A computational model

Let us develop concrete models and computational techniques for the photon complex in practical geometries.

11.1.1 Truncated Deligne representatives

For numerical and constructive purposes we work with truncated Deligne cocycles on a triangulation $\mathcal{T}$ of M. A degree-k differential character is represented by a collection of simplicial cochains together with discrete differential form data on simplices satisfying compatibility conditions mirroring the continuous photon complex.

11.1.2 Discrete curvature and integrality checks

Given discrete approximations of F_A and F_B (via Whitney forms or piecewise polynomial forms), the integrality of $\int_\sigma \mathcal{C}$ over top-dimensional simplices σ can be checked numerically. Convergence theorems for compatible finite-element methods (see Brezzi–Douglas–Marini style frameworks) guarantee that these discrete integrals converge to the continuous periods under mesh refinement.

Proposition 11.1 (Convergence of discrete periods)**.** *Let $\{\mathcal{T}_h\}$ be a sequence of shape-regular triangulations with mesh size $h \to 0$ and let discrete approximations $\mathcal{C}_h$ converge in L^1 to $\mathcal{C}$. Then for each simplex σ, $\int_\sigma \mathcal{C}_h \to \int_\sigma \mathcal{C}$. In particular global integrality checks stabilize for sufficiently fine meshes.*

11.2 Algorithmic sketch

An explicit algorithm for numerical verification:

1. Triangulate M and choose compatible finite-element spaces for forms up to degree r.
2. Interpolate A, B and κ_k to discrete degrees of freedom; assemble discrete $\mathcal{C}_h$.
3. Compute discrete integrals over chains representing generating cycles of $H_n(M;\mathbb{Z})$.
4. Verify integrality within numerical tolerance; refine mesh until stable.

12 Stacky Sheaves, Photons and Cohomological Dualities

12.1 Higher stacks as organizational structure

We outline a higher-stack viewpoint in which photonic bundles and gerbes are organized as objects in a stack $\mathscr{P}h$ over the site of smooth manifolds. Morphisms in this stack encode gauge transformations, higher gauge transformations, and coupling-preserving homotopies. This perspective clarifies naturality properties and the behavior under pullback.

12.2 Dualities and categorical correspondences

Within the stacky framework one can formulate duality functors exchanging electric and magnetic data. Concretely, there is an involutive operation on the stack exchanging a gerbe with curvature H and a dual object whose curvature corresponds to the Poincaré dual cycle supporting the electric sector. When extended along moduli of backgrounds this gives rise to correspondences reminiscent of (but not identical to) Geometric Langlands dualities.

Conjecture 12.1 (Categorical photonic duality)**.** *There exists a pair of adjoint functors between suitable derived categories of modules over the stack $\mathscr{P}h$ and its dual $\mathscr{P}h^\vee$ which exchange gerbe-flux data with dual connection data. These functors preserve photon characteristic classes up to natural equivalence.*

12.3 Implications and outlook

The stacky viewpoint suggests new invariants obtained by applying higher-categorical traces (e.g. Hochschild-type invariants) to categories of photonic twisted sheaves. Physically, such categorical invariants may control stability of multi-channel entanglement phenomena and global dualities between different photonic phases.

13 Conclusion and open directions

Photon Cohomology $H^\bullet_{\mathrm{ph}}(M)$ provides a natural differential cohomology framework capturing higher gerbe data and coupling-induced photonic invariants. The characteristic class c_{ph} encodes obstruction to trivializing exotic photonic channels and bridges geometric topology with measurable photonics phenomena. Future work includes: (i) rigorous development of categorical formulations and Langlands-like dualities for photonic sectors; (ii) numerical implementations on realistic photonic crystal geometries; (iii) experimental proposals to measure higher-holonomy effects.